Code No : **20SH1T01**

R20

I B. Tech I Semester Regular Examinations, August - 2021
MATHEMATICS - I
(Common to ALL Branches)

Time : 3 Hours **Max. Marks : 70**

Note : Answer **ONE** question from each unit **(5 × 14 = 70 Marks)**

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UNIT-I

1. a) Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ [7M]

   b) Show that the family of curves $y^2 = 4a(x + a)$ is self- orthogonal. [7M]

(OR)

2. a) Solve $(xysinxy + cosxy)y\,dx + (xysinxy - cosxy)x\,dy = 0$ [7M]

   b) A body kept in air with temperature 25^0 C, cools from 140^0 C to 80^0 C in 20 minutes. Find when the body cools down to 35^0 C. [7M]

UNIT-II

3. a) Solve $(D^2 - 4D + 4)y = e^{3x} Cos\ 2x$ [7M]

   b) Solve $\frac{d^2y}{dx^2} + 4y = Sec\ 2x$ using the method of variation of parameters. [7M]

(OR)

4. a) Solve $(D^2 + 3D + 2)y = xe^x sinx$ [7M]

   b) An RCL circuit connected in series has $R = 10\ Ohms$, $C = 10^{-2} Farad, L = \frac{1}{2} henries$ and an applied Voltage $E(t) = 12\ volts$. Assuming no initial current and no initial charge at $t = 0$ when the voltage is first applied, find the subsequent current in the system. [7M]

UNIT-III

5. a) State and verify Rolle's theorem for the function $f(x) = cos\frac{1}{x}$ in the interval $[-1,1]$ [7M]

   b) Use Lagrange's mean value theorem show that [7M]

   $$\frac{x}{\sqrt{1-x^2}} \leq sin^{-1}x < x \text{ if } 0 \leq x < 1$$

(OR)

6. a) Verify Cauchy's mean value theorem for $f(x) = \sin x$ and $g(x) = \cos x$ in the interval $[a, b]$ [7M]

   b) Expand $tan\,x$ by Maclaurin's series upto the term containing x^5 [7M]
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UNIT-IV

7. a) Expand $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ upto third degree terms. [7M]

b) Find the maxima and minima of the function [7M]

$$f(x,y) = x^3y^2(1-x-y)$$

(OR)

8. a) If $u = \log\,(x^3+y^3+z^3-3xyz)$, [7M]

show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

b) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ [7M]

UNIT-V

9. a) Evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ by changing the order of integration. [7M]

b) Evaluate $\int_0^{\log 2}\int_0^x\int_0^{x+logy} e^{x+y+z}dz\,dy\,dx$ [7M]

(OR)

10. a) Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\,dx\,dy$ by changing the order of integration. [7M]

b) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dxdy$ by changing to polar coordinates. [7M]

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