Code No: **20SH3T01** **R20**

II B. TECH I SEMESTER REGULAR EXAMINATIONS, FEB-2022
MATHEMATICS - III
(Common to ALL BRANCHES)

Time: 3 Hours **Max. Marks: 70**

Note : Answer **ONE** question from each unit **(5 × 14 = 70 Marks)**

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UNIT-I

1. a) Find non-singular matrices P and Q such that PAQ is in the normal form of A for the matrix, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ [7M]

   b) Find Eigen values and Eigen vectors of following matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. [7M]

(OR)

2. a) Check the consistency of following system of equations [7M]
   $x + 2y + 2z = 5$; $2x + y + 3z = 6$;
   $3x - y + 2z = 4$; $x + y + z = -1$. If consistent solve them.

   b) Verify that the eigen values of A^2 and A^{-1} are respectively the squares and reciprocals of the eigenvalues of A, given that $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ [7M]

UNIT-II

3. Verify Cayley-Hamilton theorem for the matrix [14M]
   $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ also determine A^{-1} and A^4.

(OR)

4. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ to the sum of squares form and find the corresponding orthogonal transformation. Also indicate the nature, index and signature of the quadratic form. [14M]
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UNIT-III

5. a) Determine the constants a and b such that the curl of $(2xy+3yz)\bar{i}+(x^2+axz-4z^2)\bar{j}+(3xy+2byz)\bar{k}$ is zero. [7M]

b) Prove that $\nabla^2\left[\nabla.\left(\frac{\bar{r}}{r^2}\right)\right]=2r^{-4}$, where $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$. [7M]

(OR)

6. a) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q is the point (5, 0, 4) In what direction it will be maximum?. Find the maximum value of it. [7M]

b) Prove that $curl\left[(\bar{r}\times\bar{a})\times\bar{b}\right]=\bar{b}\times\bar{a}$, where $\bar{a}$ and $\bar{b}$ are constants. [7M]

UNIT-IV

7. a) Evaluate $\iiint_V \bar{F}\,dV$ where $\bar{F}=x\bar{i}+y\bar{j}+2z\bar{k}$ and V is the volume enclosed by the planes $x = 0$, $x = b$, $y = 0$, $y = a$, $z = b^2$ and the surface $z = x^2$. [7M]

b) Evaluate $\iint_S \bar{F}\cdot\bar{n}\,ds$ using Gauss divergence theorem where $\bar{F}=2xy\bar{i}+yz^2\bar{j}+zx\bar{k}$ and S is the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $y = 3$, $x + 2z = 6$. [7M]

(OR)

8. a) Evaluate $\int_C \bar{F}.d\bar{r}$ along the curve $x^2 + y^2 = 1$, $z = 1$ in the positive direction from (0, 1, 1) to (1, 0, 1), where $\bar{F}=(yz+2x)\bar{i}+xz\bar{j}+(xy+2z)\bar{k}$. [7M]

b) By Green's theorem find the area of the region for $\frac{1}{2}\int_c xdy-ydx$ bounded by the parabola $y = x^2$ and the line $y = x + 2$. [7M]

UNIT-V

9. a) Find the solution of $(D^2+2DD'+D'^2-2D-2D')z=\sin(x+2y)$ [7M]

b) Solve $9(p^2z+q^2)=4$ [7M]

(OR)

10. a) Find the general solution of $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$ [7M]

b) Solve $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}=e^{x+2y}+4\sin(x+y)$ [7M]

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